Supplementary document for “Adaptive Operator Selection with Bandits for Multiobjective Evolutionary Algorithm Based on Decomposition”

Ke Li, Álvaro Fialho, Sam Kwong, Qingfu Zhang

Abstract

This supplementary document provides the numerical values of the simulations with MOEA/D-DE, MOEA/D-DRA, ENS-MOEA/D and MOEA/D-FRRMAB in Table 1 and Table 2. Moreover, Fig. 1 to Fig. 10 show the performances of MOEA/D-FRRMAB on IGD with 72 different combinations of C, D and W. Fig. 11 to Fig. 20 present the performances of MOEA/D-FRRMAB on IH with 72 different combinations of C, D and W.

Table 1: Comparative results of MOEA/D-DE, MOEA/D-DRA, ENS-MOEA/D and MOEA/D-FRRMAB on IGD metric

<table>
<thead>
<tr>
<th>Problem</th>
<th>MOEA/D</th>
<th>DRA</th>
<th>ENS</th>
<th>FRRMAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>UF1</td>
<td>1.258E-3 (1.07E-4)†</td>
<td>1.589E-3 (6.50E-4)†</td>
<td>1.442E-3 (1.26E-4)†</td>
<td>1.021E-3 (1.76E-4)</td>
</tr>
<tr>
<td>UF2</td>
<td>5.710E-3 (1.55E-3)†</td>
<td>4.203E-3 (2.00E-3)†</td>
<td>3.910E-3 (1.11E-3)†</td>
<td>1.851E-3 (5.38E-4)</td>
</tr>
<tr>
<td>UF3</td>
<td>1.342E-2 (1.47E-2)†</td>
<td>4.859E-3 (5.95E-3)</td>
<td>3.820E-3 (1.45E-3)†</td>
<td>4.404E-3 (7.44E-3)</td>
</tr>
<tr>
<td>UF4</td>
<td>5.621E-2 (3.37E-3)†</td>
<td>5.963E-2 (4.26E-3)†</td>
<td>5.532E-2 (3.98E-3)†</td>
<td>5.276E-2 (3.19E-3)</td>
</tr>
<tr>
<td>UF5</td>
<td>3.152E-1 (4.85E-2)†</td>
<td>2.960E-1 (6.56E-2)</td>
<td>2.968E-1 (1.34E-1)</td>
<td>2.949E-1 (4.61E-2)</td>
</tr>
<tr>
<td>UF6</td>
<td>1.026E-1 (1.01E-1)†</td>
<td>1.686E-1 (1.44E-1)†</td>
<td>9.810E-2 (7.23E-2)†</td>
<td>8.298E-2 (5.65E-2)</td>
</tr>
<tr>
<td>UF7</td>
<td>1.593E-3 (5.03E-4)†</td>
<td>2.918E-3 (4.09E-3)†</td>
<td>2.121E-3 (3.57E-4)†</td>
<td>1.202E-3 (2.49E-4)</td>
</tr>
<tr>
<td>UF8</td>
<td>5.760E-2 (8.94E-3)†</td>
<td>4.779E-2 (1.05E-2)†</td>
<td>4.299E-2 (6.00E-3)†</td>
<td>4.067E-2 (3.54E-3)</td>
</tr>
<tr>
<td>UF9</td>
<td>5.268E-2 (4.02E-2)†</td>
<td>1.052E-1 (5.13E-2)†</td>
<td>5.271E-2 (3.99E-2)†</td>
<td>3.826E-2 (3.49E-2)</td>
</tr>
<tr>
<td>UF10</td>
<td>5.379E-1 (6.68E-2)†</td>
<td>4.138E-1 (7.01E-2)†</td>
<td>3.989E-1 (8.63E-2)†</td>
<td>5.266E-1 (7.14E-2)</td>
</tr>
</tbody>
</table>

Wilcoxon’s rank sum test at a 0.05 significance level is performed between MOEA/D-FRRMAB and each of MOEA/D-DE, MOEA/D-DRA and ENS-MOEA/D. † and ‡ denote the performance of the corresponding algorithm is significantly worse than and better than that of the proposed MOEA/D-FRRMAB, respectively. The best mean metric value is highlighted in boldface with gray background.
Table 2: Comparative results of MOEA/D-DE, MOEA/D-DRA, ENS-MOEA/D and MOEA/D-FRRMAB on $I_H$ metric

<table>
<thead>
<tr>
<th>Problem</th>
<th>MOEA/D</th>
<th>DRA</th>
<th>ENS</th>
<th>FRRMAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>UF1</td>
<td>3.6601(1.30E-3)</td>
<td>3.6588(2.83E-3)</td>
<td>3.6599(1.01E-3)</td>
<td>3.6625(2.05E-3)</td>
</tr>
<tr>
<td>UF2</td>
<td>3.6464(1.37E-2)</td>
<td>3.6479(1.43E-2)</td>
<td>3.6501(5.18E-3)</td>
<td>3.6554(1.11E-2)</td>
</tr>
<tr>
<td>UF3</td>
<td>3.6163(8.15E-2)†</td>
<td>3.6490(3.46E-2)</td>
<td>3.6599(1.65E-3)†</td>
<td>3.6543(3.53E-2)</td>
</tr>
<tr>
<td>UF4</td>
<td>3.1680(1.62E-2)†</td>
<td>3.1601(1.95E-2)†</td>
<td>3.1692(1.90E-2)†</td>
<td>3.1825(1.20E-2)</td>
</tr>
<tr>
<td>UF5</td>
<td>2.6478(1.57E-1)†</td>
<td>2.7014(2.99E-1)</td>
<td>2.6573(3.04E-1)</td>
<td>2.7218(1.36E-1)</td>
</tr>
<tr>
<td>UF6</td>
<td>3.1008(2.69E-1)†</td>
<td>2.9023(3.33E-1)†</td>
<td>3.1105(2.14E-1)†</td>
<td>3.1373(1.82E-1)</td>
</tr>
<tr>
<td>UF7</td>
<td>3.4901(5.83E-3)</td>
<td>3.4768(4.57E-2)</td>
<td>3.4876(2.08E-3)†</td>
<td>3.4902(4.83E-3)</td>
</tr>
<tr>
<td>UF8</td>
<td>7.3320(1.84E-2)†</td>
<td>7.3659(2.09E-2)†</td>
<td>7.3700(1.34E-2)</td>
<td>7.3715(1.32E-2)</td>
</tr>
<tr>
<td>UF9</td>
<td>7.5512(1.73E-1)†</td>
<td>7.3549(2.38E-1)†</td>
<td>7.5494(1.74E-1)†</td>
<td>7.6437(1.53E-1)</td>
</tr>
<tr>
<td>UF10</td>
<td>3.4193(3.00E-1)†</td>
<td>3.7542(3.27E-1)†</td>
<td>3.8276(6.50E-1)†</td>
<td>3.5095(3.45E-1)</td>
</tr>
</tbody>
</table>

Wilcoxon’s rank sum test at a 0.05 significance level is performed between MOEA/D-FRRMAB and each of MOEA/D-DE, MOEA/D-DRA and ENS-MOEA/D. † and ‡ denote the performance of the corresponding algorithm is significantly worse than and better than that of the proposed MOEA/D-FRRMAB, respectively. And the best mean metric value is highlighted in boldface with gray background.
Figure 1: Median IGD metric values found by MOEA/D-FRRMAB with 72 different combinations of C, D and W on UF 1 (a to c)

Figure 2: Median IGD metric values found by MOEA/D-FRRMAB with 72 different combinations of C, D and W on UF 2 (a to c)
Figure 3: Median $IGD$ metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 3 (a to c).

Figure 4: Median $IGD$ metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 4 (a to c).

Figure 5: Median $IGD$ metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 5 (a to c).
Figure 6: Median IGD metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 6 (a to c)

Figure 7: Median IGD metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 7 (a to c)

Figure 8: Median IGD metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 8 (a to c)
Figure 9: Median $IGD$ metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 9 (a to c)

Figure 10: Median $IGD$ metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 10 (a to c)
Figure 11: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 1 (a to c)

Figure 12: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 2 (a to c)
Figure 13: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 3 (a to c)

Figure 14: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 4 (a to c)

Figure 15: Median $I_H$ metric values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 5 (a to c)
Figure 16: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 6 (a to c)

Figure 17: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 7 (a to c)

Figure 18: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 8 (a to c)
Figure 19: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 9 (a to c)

Figure 20: Median $I_H$ values found by MOEA/D-FRRMAB with 72 different combinations of $C$, $D$ and $W$ on UF 10 (a to c)