Supplementary Document of "Interactive Decomposition Multi-Objective Optimisation via Progressively Learned Value Functions"

Ke Li#1, Renzhi Chen#2, Dragan Savić[3] and Xin Yao[2,3]

[1]Department of Computer Science, University of Exeter
[2]Department of Computer Science and Engineering, Southern University of Science and Technology
[3]CERCIA, School of Computer Science, University of Birmingham
[4]Department of Engineering, University of Exeter
*Email: {k.li, d.savic}@exeter.ac.uk, rxc332@cs.bham.ac.uk, xiny@sustc.edu.cn
#The first two authors make equal contributions to this paper.

1 Description of Benchmark Suite

This section provides the formal definitions and characteristics of the test problems used in our empirical studies, i.e., DTLZ [1] test problems.

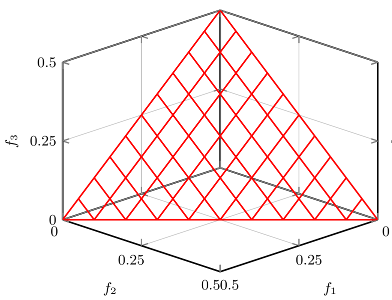
DTLZ1 is mathematically defined as:

$$\left.\begin{aligned} \min f_1(\mathbf{x}) &= \frac{1}{2}x_1x_2\cdots x_{m-1}(1+g(\mathbf{x}_m)),\\ \min f_2(\mathbf{x}) &= \frac{1}{2}x_1x_2\cdots(1-x_{m-1})(1+g(\mathbf{x}_m)),\\ &\vdots\\ \min f_{m-1}(\mathbf{x}) &= \frac{1}{2}x_1(1-x_2)(1+g(\mathbf{x}_m)),\\ \min f_m(\mathbf{x}) &= \frac{1}{2}(1-x_1)(1+g(\mathbf{x}_m)) \end{aligned}\right\} \tag{1}$$

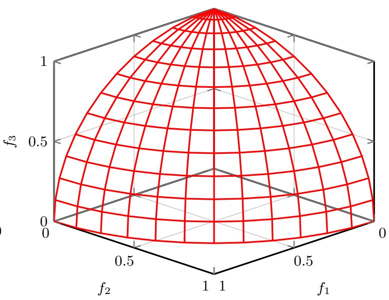
where

$$g(\mathbf{x}_m) = 100\Big[|\mathbf{x}_m| + \sum_{x_i\in\mathbf{x}_m}(x_i-0.5)^2 - \cos(20\pi(x_i-0.5))\Big], \tag{2}$$

$\mathbf{x}_m = (x_m, \cdots, x_n)^T$ and $x_i \in [0,1]$. The Pareto front (PF) of DTLZ1 is a hyper-plane that intersects each coordinates at 0.5. Fig. 1(a) gives an example in the three-objective case. The search space of DTLZ1 contains 11^k-1, where $k = n-m+1$, local PFs. In particular, we set $k = 5$ in our empirical studies. Note that the multi-modality hinders an EMO algorithm to converge to the global PF.



(a) DTLZ1



(b) DTLZ2 to DTLZ4

Figure 1: PF of DTLZ test problems.

DTLZ2 is mathematically defined as:

$$
\left.\begin{aligned}
\min f_1(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\cos(x_{m-2}\pi/2) \\
\cos(x_{m-1}\pi/2), \\
\min f_2(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\cos(x_{m-2}\pi/2) \\
\sin(x_{m-1}\pi/2), \\
\min f_3(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1\pi/2)\cdots\sin(x_{m-2}\pi/2), \\
&\vdots \\
\min f_m(\mathbf{x}) &= (1+g(\mathbf{x}_m))\sin(x_1\pi/2),
\end{aligned}\right\} \tag{3}
$$

where

$$
g(\mathbf{x}_m) = \sum_{x_i\in\mathbf{x}_m}(x_i-0.5)^2. \tag{4}
$$

$\mathbf{x}_m = (x_m, \cdots, x_n)^T$ and $x_i \in [0,1]$. Once again, DTLZ2 also has $k = n - m + 1$ and $k = 10$ this time. The PF of DTLZ2 is a unit sphere inside the first octant. Fig. 1(b) gives an example in the three-objective case.

DTLZ3 has the same objective functions as DTLZ2, but the $g(\mathbf{x}_m)$ is the same as equation (2). In this case, DTLZ3 also has a multi-modality property.

DTLZ4 is mathematically defined as:

$$
\left.\begin{aligned}
\min f_1(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1^\alpha\pi/2)\cdots\cos(x_{m-2}^\alpha\pi/2) \\
\cos(x_{m-1}^\alpha\pi/2), \\
\min f_2(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1^\alpha\pi/2)\cdots\cos(x_{m-2}^\alpha\pi/2) \\
\sin(x_{m-1}^\alpha\pi/2), \\
\min f_3(\mathbf{x}) &= (1+g(\mathbf{x}_m))\cos(x_1^\alpha\pi/2)\cdots\sin(x_{m-2}^\alpha\pi/2), \\
&\vdots \\
\min f_m(\mathbf{x}) &= (1+g(\mathbf{x}_m))\sin(x_1^\alpha\pi/2),
\end{aligned}\right\} \tag{5}
$$

where the g function is the same as defined in equation (4). The PF of DTLZ4 is the same as DTLZ2, but the search space has a biased density toward particular coordinates.

2 Settings of the DM's Preference Information

In this section, we provide the DM's preference information in terms of the utopia weights $\mathbf{w}^*$ and their corresponding Pareto-optimal optimal solution (also known as the DM's "golden" point) in Table 1 to Table 4.

Table 1: Settings of the utopia weights $\mathbf{w}^*$ that prefers the middle of the PF.

	objective index									
m	1	2	3	4	5	6	7	8	9	10
3	0.4	0.3	0.3	\	\	\	\	\	\	\
5	0.2	0.18	0.24	0.18	0.2	\	\	\	\	\
8	0.13	0.13	0.12	0.13	0.14	0.12	0.11	0.11	\	\
10	0.11	0.12	0.11	0.08	0.1	0.09	0.1	0.09	0.11	0.09

Table 2: Settings of the corresponding DM's "golden" point with respect to the $\mathbf{w}^*$ that prefers the middle of the PF.

		objective index									
m	problem	1	2	3	4	5	6	7	8	9	10
3	DTLZ1	0.200	0.150	0.150	\	\	\	\	\	\	\
	DTLZ2-4	0.686	0.514	0.514	\	\	\	\	\	\	\
5	DTLZ1	0.100	0.090	0.120	0.090	0.100	\	\	\	\	\
	DTLZ2-4	0.445	0.400	0.533	0.400	0.445	\	\	\	\	\
8	DTLZ1	0.065	0.065	0.060	0.065	0.070	0.060	0.055	0.055	\	\
	DTLZ2-4	0.370	0.370	0.342	0.370	0.399	0.342	0.313	0.313	\	\
10	DTLZ1	0.055	0.060	0.055	0.040	0.050	0.045	0.050	0.045	0.055	0.045
	DTLZ2-4	0.345	0.377	0.345	0.251	0.314	0.283	0.314	0.283	0.345	0.283

Table 3: Settings of the utopia weights $\mathbf{w}^*$ that prefers one side of the PF.

		objective index									
m	problem	1	2	3	4	5	6	7	8	9	10
3	DTLZ1	0.2	0.2	0.6	\	\	\	\	\	\	\
	DTLZ2-4	0.2	0.6	0.2	\	\	\	\	\	\	\
5	DTLZ1	0.1	0.09	0.08	0.08	0.65	\	\	\	\	\
	DTLZ2-4	0.08	0.65	0.1	0.09	0.08	\	\	\	\	\
8	DTLZ1	0.5	0.07	0.08	0.07	0.07	0.06	0.08	0.07	\	\
	DTLZ2-4	0.07	0.06	0.08	0.07	0.5	0.07	0.08	0.07	\	\
10	DTLZ1	0.05	0.06	0.05	0.06	0.06	0.5	0.05	0.06	0.05	0.06
	DTLZ2-4	0.06	0.06	0.5	0.05	0.06	0.05	0.06	0.05	0.06	0.05

Table 4: Settings of the corresponding DM's "golden" point with respect to the $\mathbf{w}^*$ that prefers one side of the PF.

		objective index									
m	problem	1	2	3	4	5	6	7	8	9	10
3	DTLZ1	0.100	0.100	0.300	╲	╲	╲	╲	╲	╲	╲
	DTLZ2-4	0.302	0.905	0.302	╲	╲	╲	╲	╲	╲	╲
5	DTLZ1	0.050	0.045	0.040	0.040	0.325	╲	╲	╲	╲	╲
	DTLZ2-4	0.119	0.965	0.149	0.134	0.119	╲	╲	╲	╲	╲
8	DTLZ1	0.250	0.035	0.040	0.035	0.035	0.030	0.040	0.035	╲	╲
	DTLZ2-4	0.131	0.112	0.150	0.131	0.935	0.131	0.150	0.131	╲	╲
10	DTLZ1	0.025	0.030	0.025	0.030	0.030	0.250	0.025	0.030	0.025	0.030
	DTLZ2-4	0.114	0.114	0.948	0.095	0.114	0.095	0.114	0.095	0.114	0.095

3 Plots of the Population Distribution

In this section, as shown in Fig. 2 to Fig. 9, we show the plots of the population (with respect to the best approximation error) obtained by different algorithms on DTLZ1 to DTLZ4 test problems. In particular, we show the results for different preference specifications, i.e., preference on the middle of the PF or on one side of the PF, in different plots.

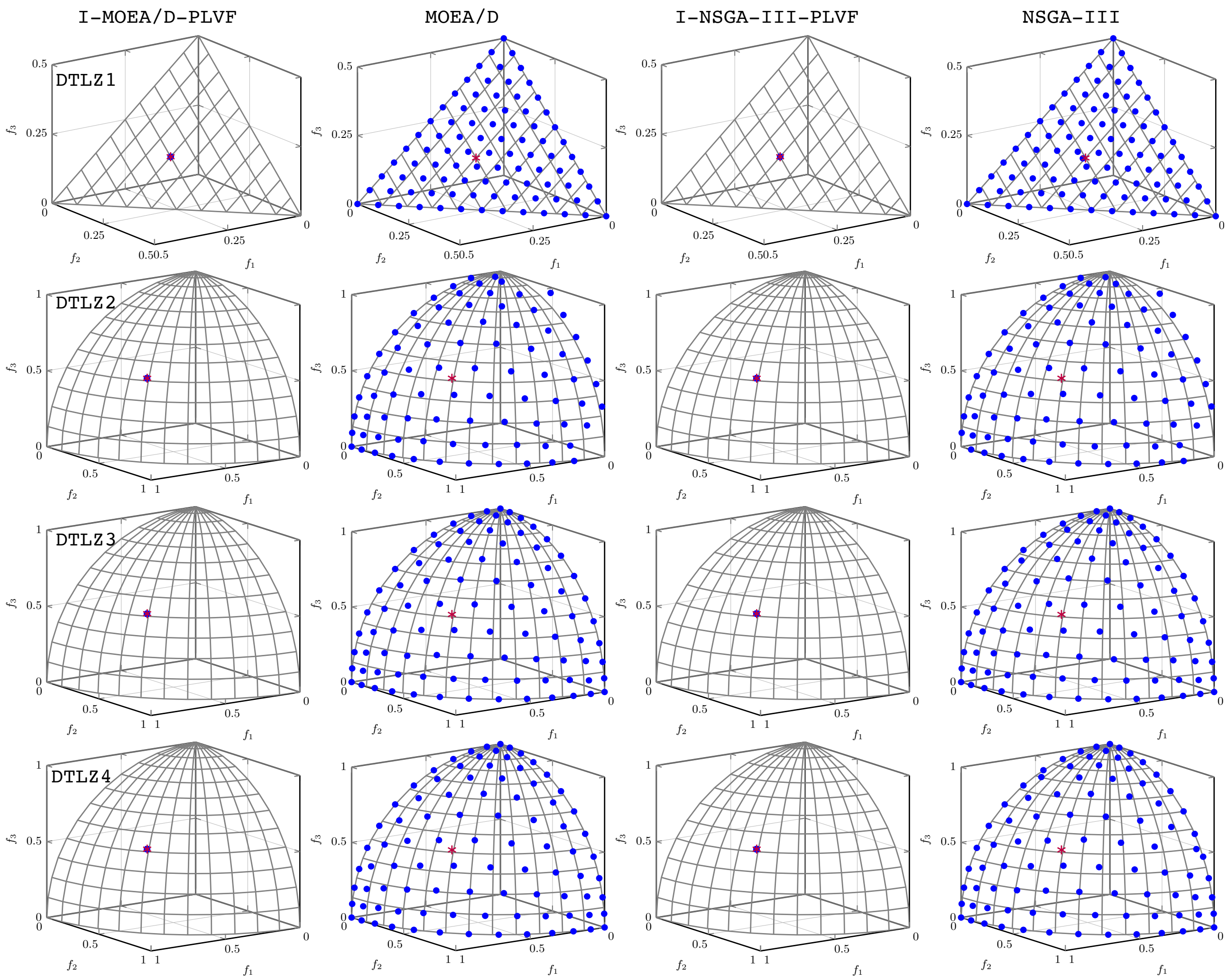


Figure 2: Solutions obtained on 3-objective DTLZ1 to DTLZ4 test problems. The DM's "golden" point, which prefers the middle region of the PF, is represented as the red asterisk.

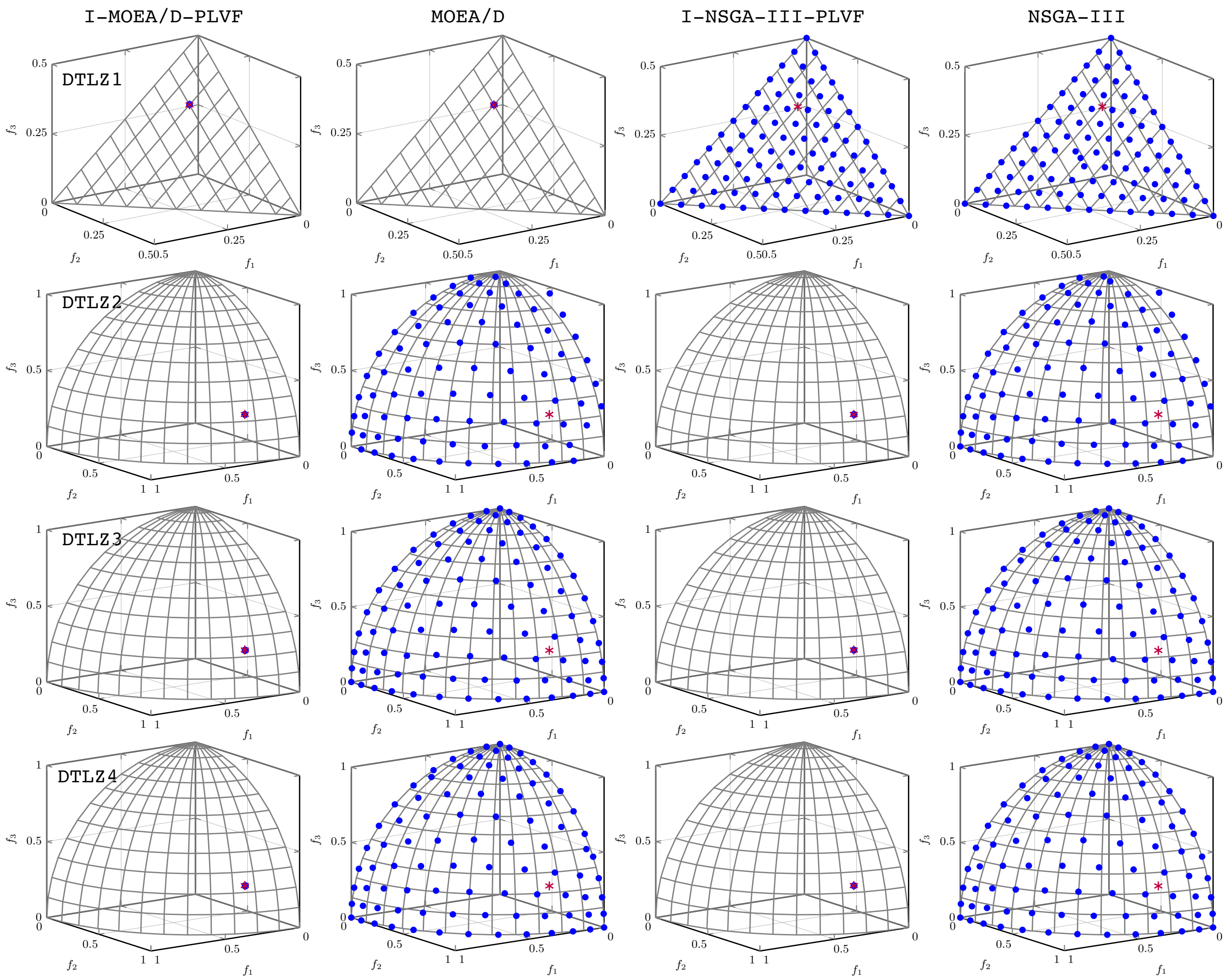


Figure 3: Solutions obtained on 3-objective DTLZ1 to DTLZ4 test problems. The DM's "golden" point, which prefers one side of the PF, is represented as the red asterisk.

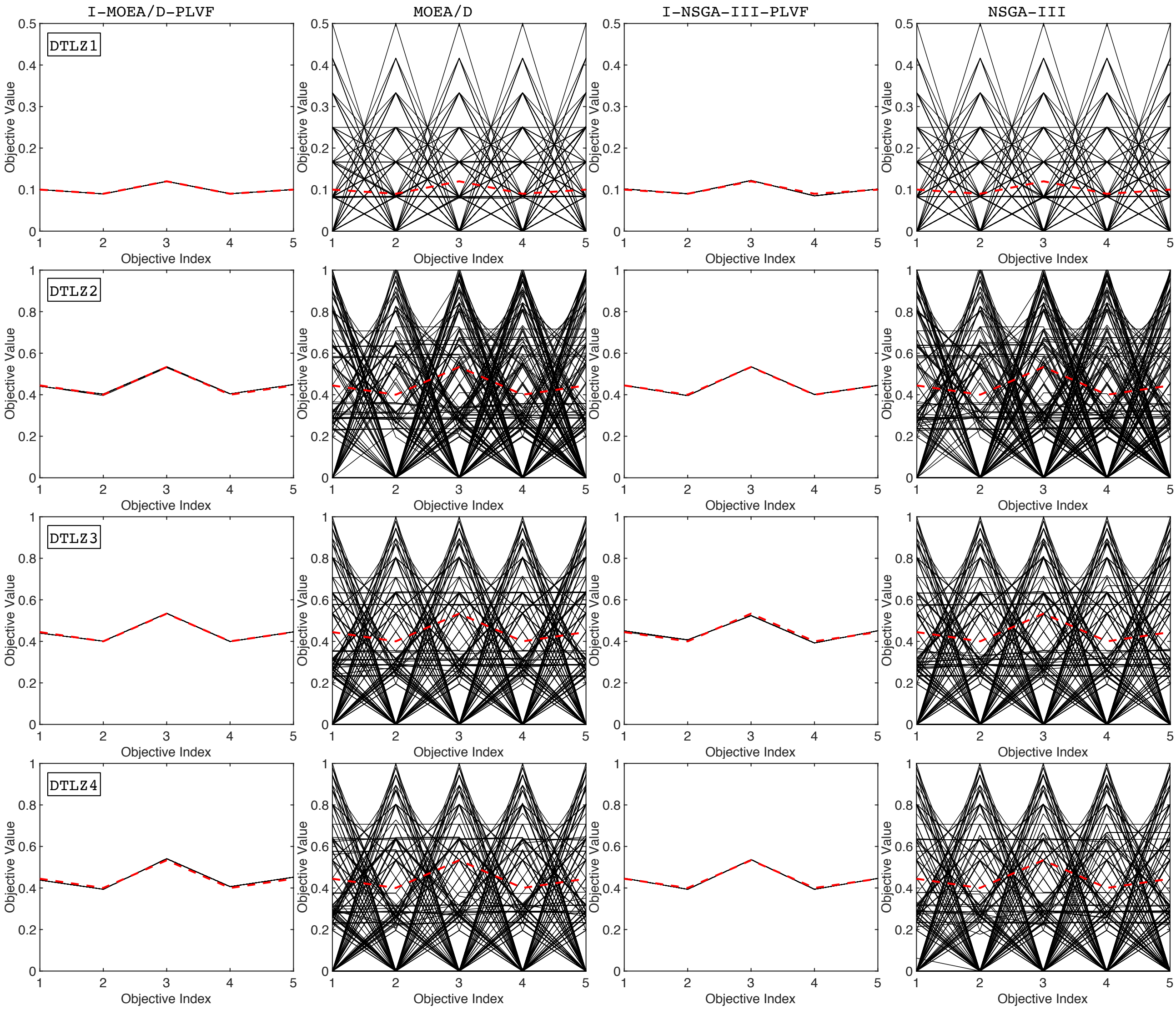


Figure 4: Solutions obtained on 5-objective DTLZ1 to DTLZ4 test problems. The DM's "golden" point, which prefers the middle region of the PF, is represented as the red dotted line.

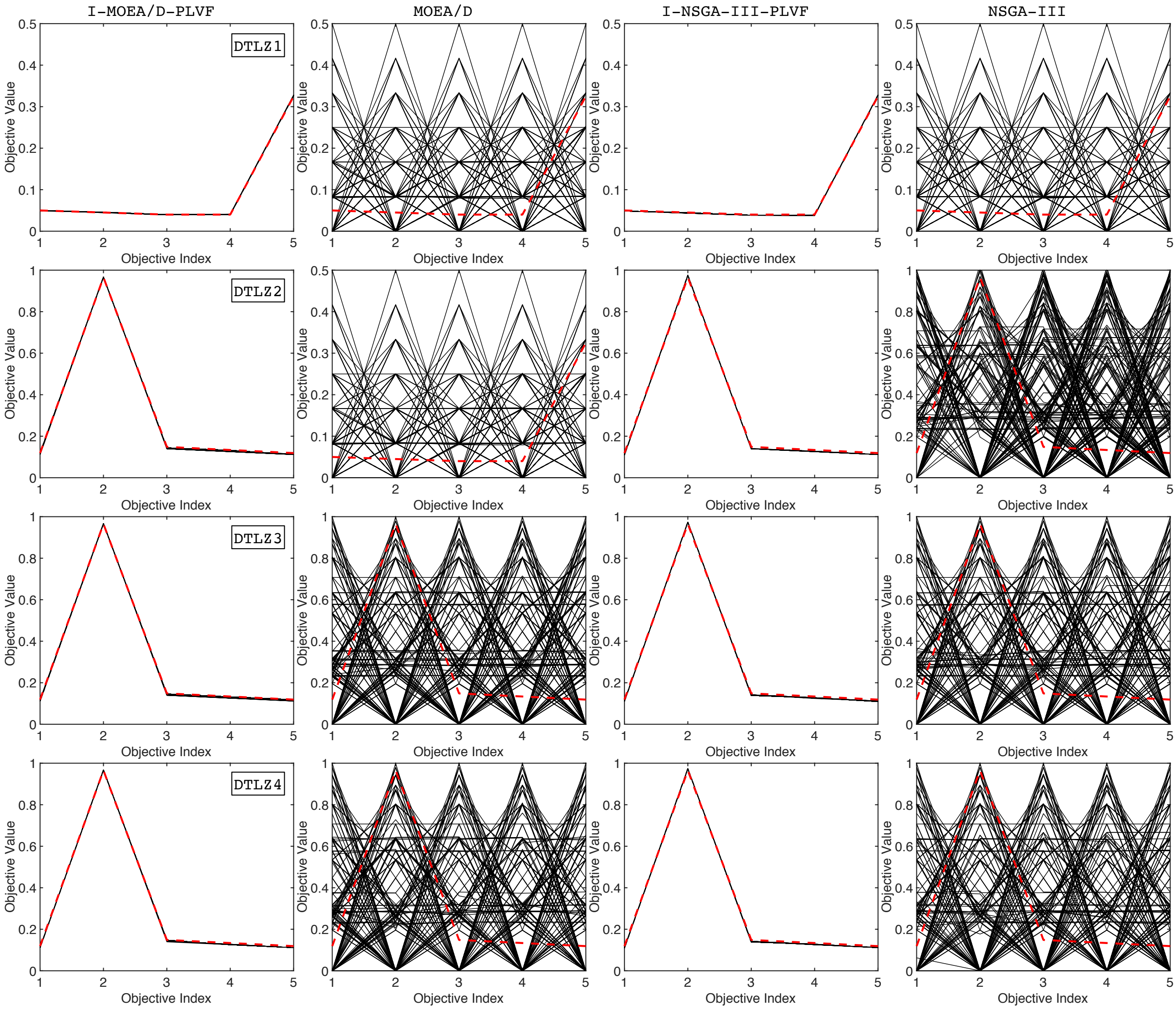


Figure 5: Solutions obtained on 5-objective DTLZ1 to DTLZ4 test problems. The DM's "golden" point, which prefers one side of the PF, is represented as the red dotted line.

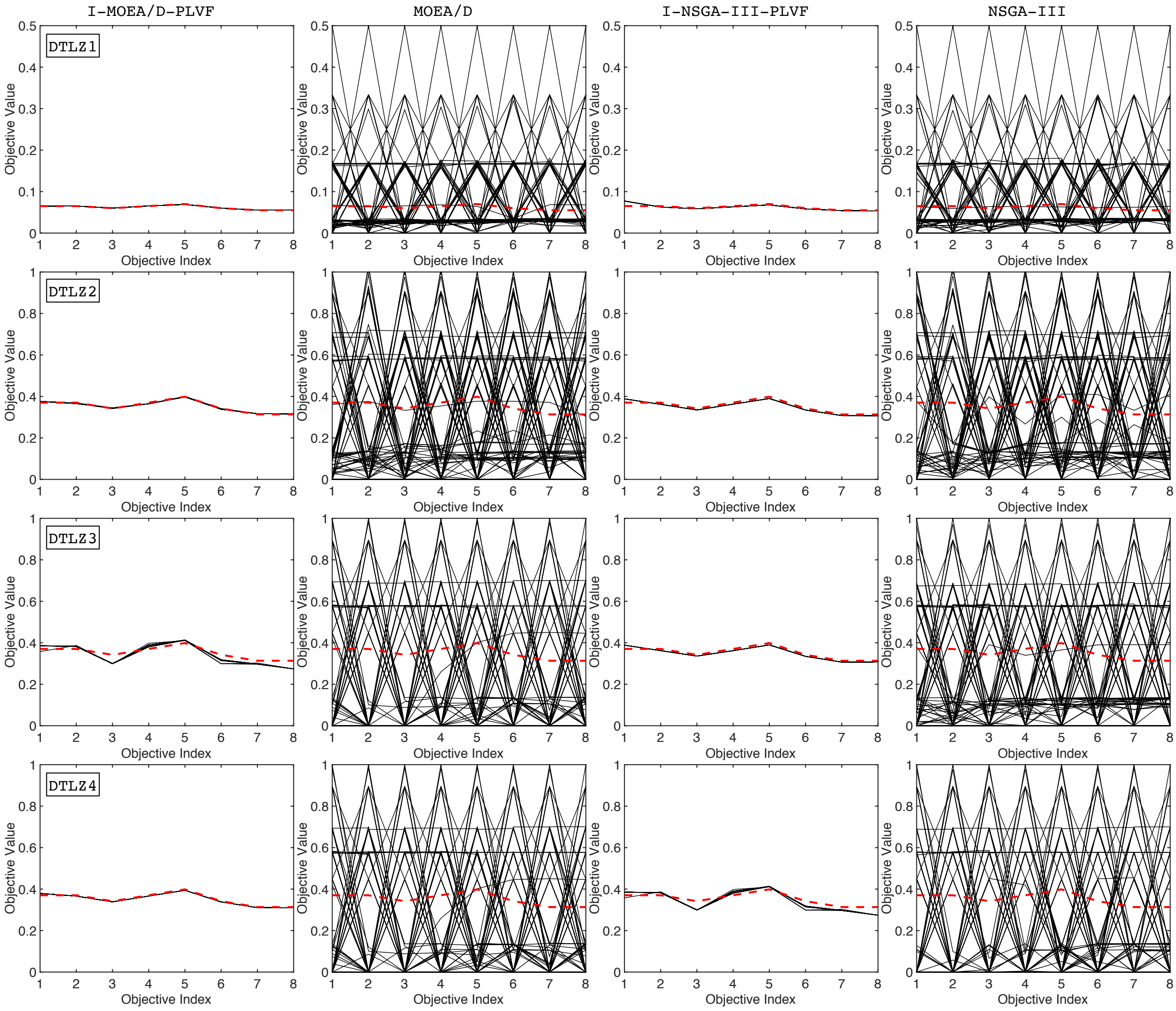


Figure 6: Solutions obtained on 8-objective DTLZ1 to DTLZ4 test problems. The DM's "golden" point, which prefers the middle region of the PF, is represented as the red dotted line.

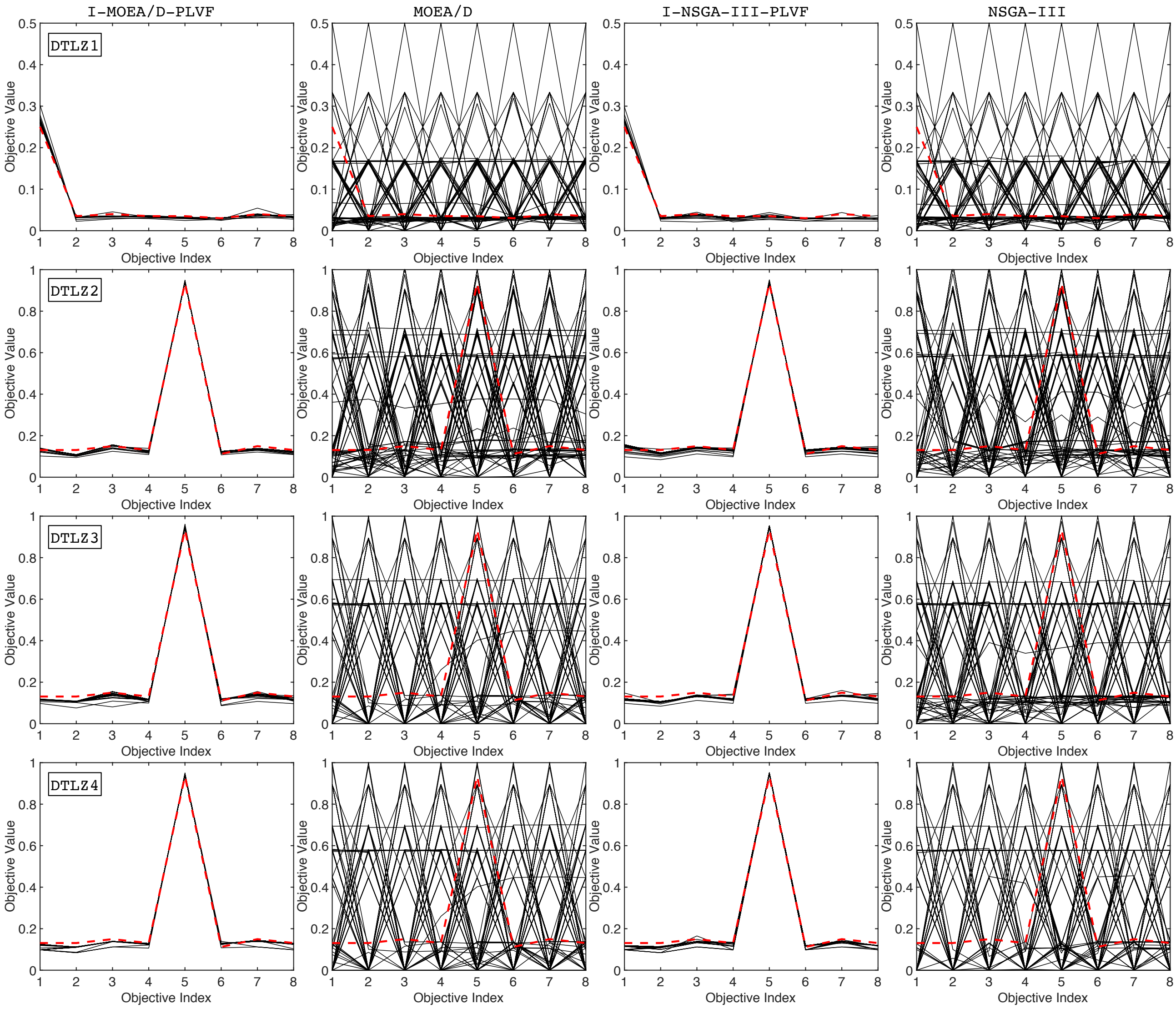


Figure 7: Solutions obtained on 8-objective DTLZ1 to DTLZ4 test problems. The DM's "golden" point, which prefers one side of the PF, is represented as the red dotted line.

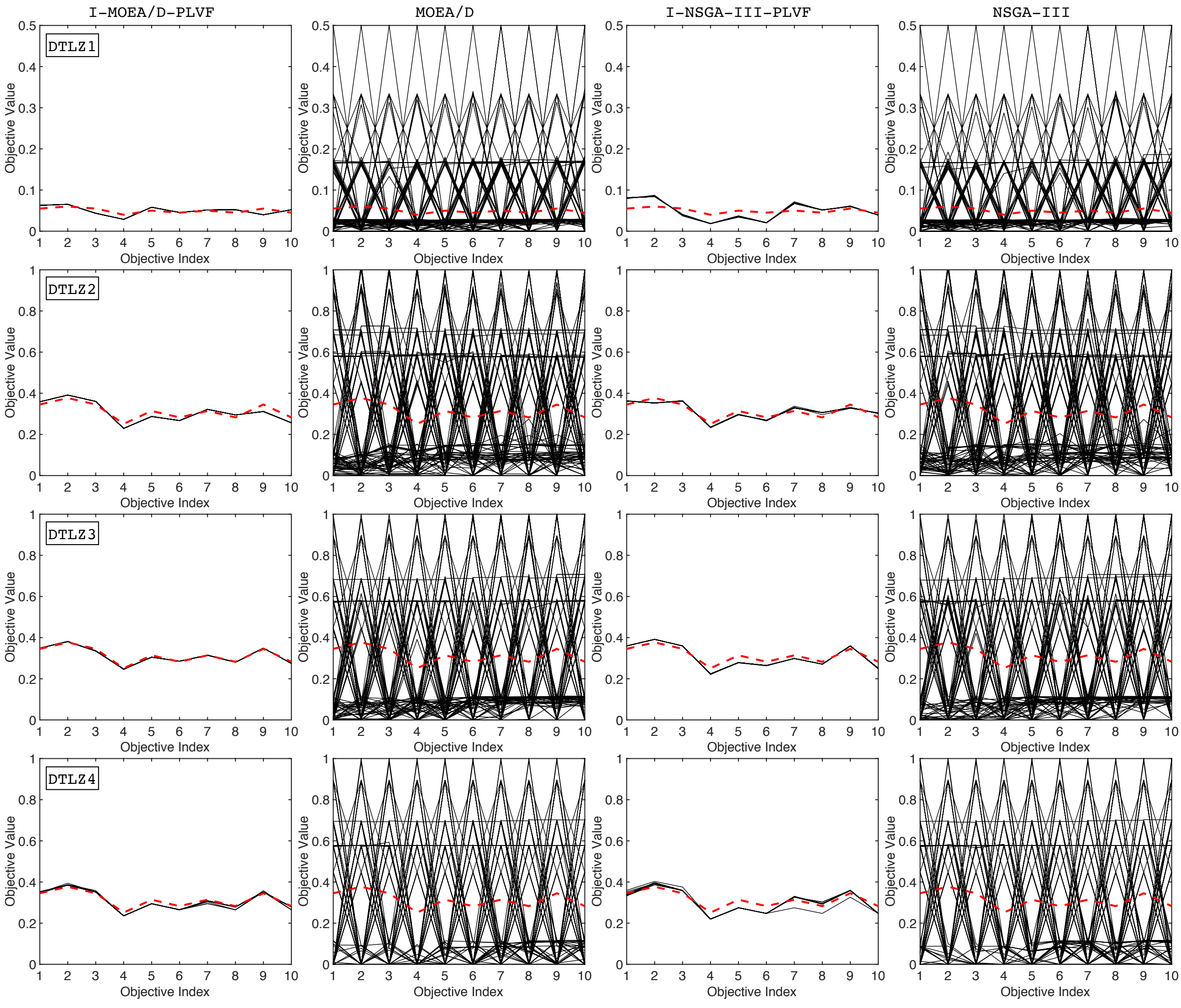


Figure 8: Solutions obtained on 10-objective DTLZ1 DTLZ4 test problems. The DM's "golden" point, which prefers the middle region of the PF, is represented as the red dotted line.

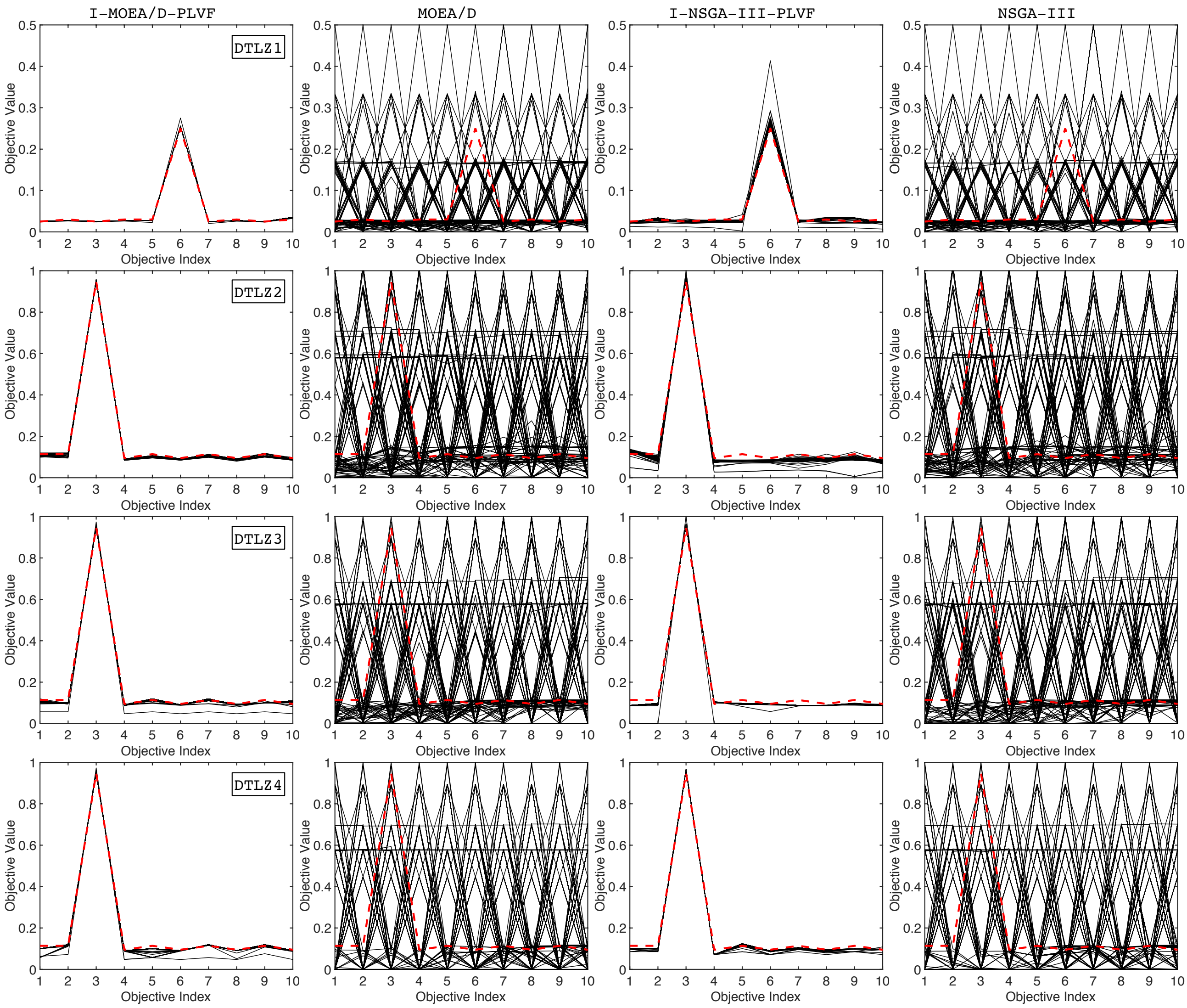


Figure 9: Solutions obtained on 10-objective DTLZ1 DTLZ4 test problems. The DM's "golden" point, which prefers one side of the PF, is represented as the red dotted line.

4 Trajectories of the Approximation Error Versus the Number of Generations

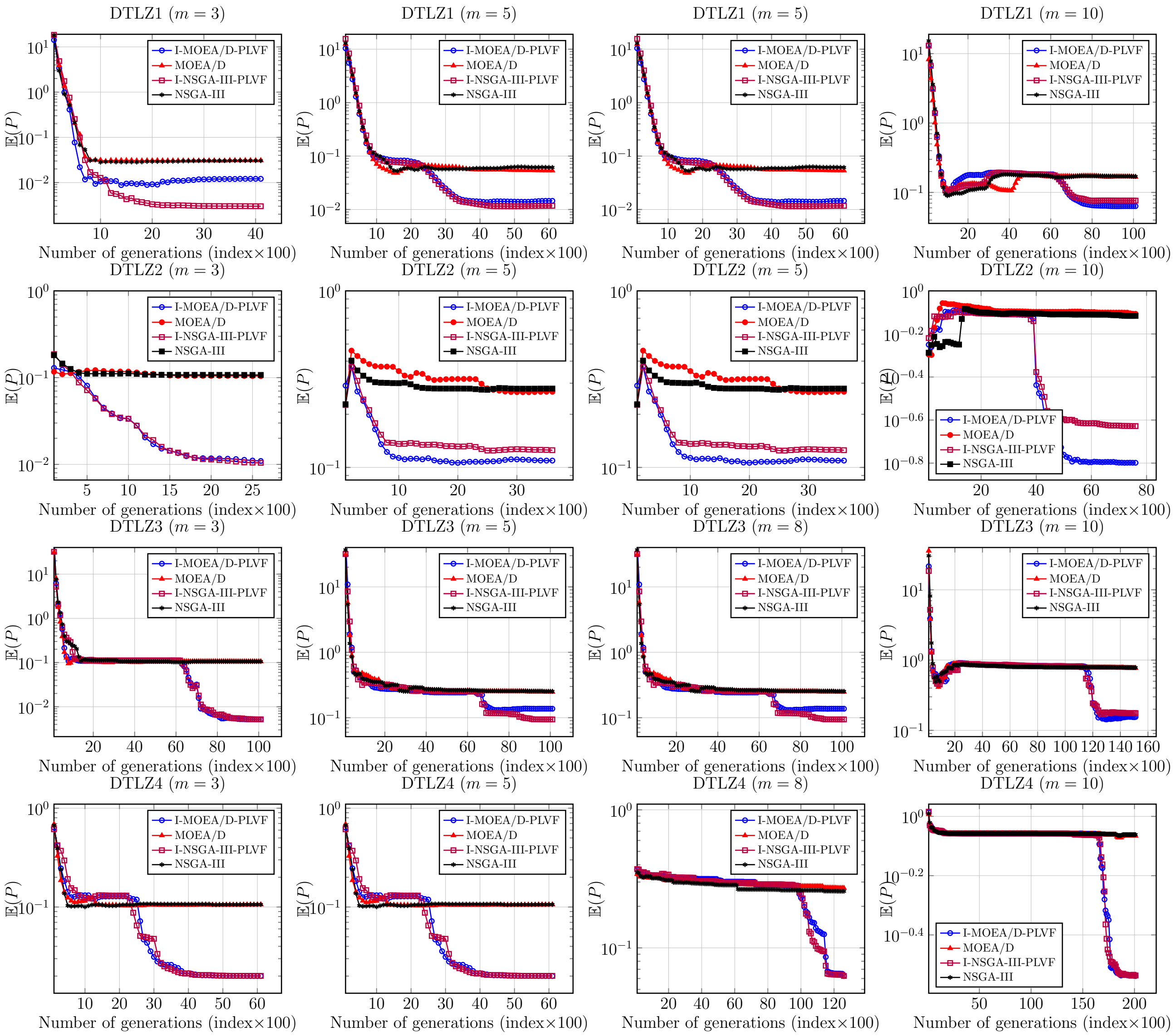


Figure 10: Trajectories of the approximation error $\mathbb{E}(P)$ versus the number of generations on DTLZ1 to DTLZ4 test problems. The DM's "golden" point prefers the middle region of the PF.

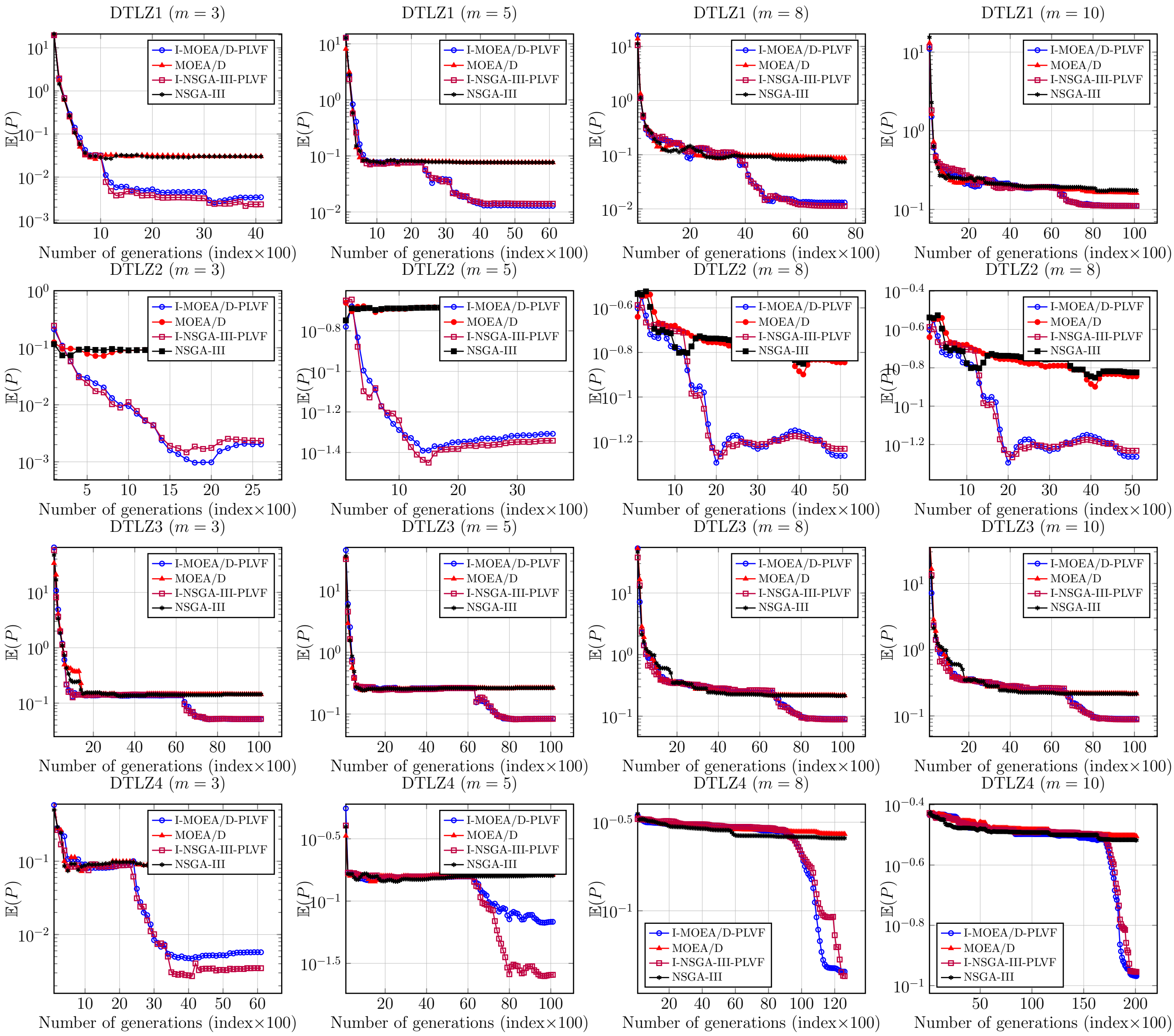


Figure 11: Trajectories of the approximation error $\mathbb{E}(P)$ versus the number of generations on DTLZ1 to DTLZ4 test problems. The DM's "golden" point prefers one side of the PF.

5 Parametric Studies

In this section, we show the empirical results of parametric studies on some parameters associated with the proposed interactive framework. Specifically, for the number of incumbent candidates presented to the DM for scoring, we set $\mu = \{5, 10, 20\}$; for the number of generations between two consecutive consultation sessions, we set $\tau = \{10, 25, 50\}$; and for the step size of the reference point update, we set $\eta = \{0.1, 0.3, 0.5, 0.7, 0.9\}$.

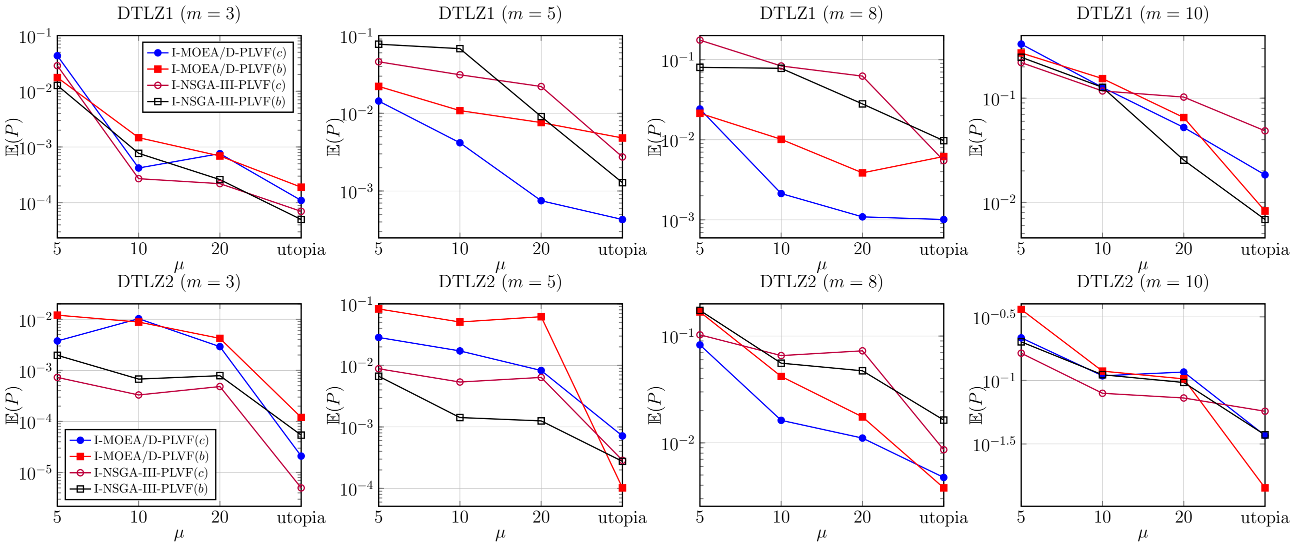


Figure 12: Variations of the approximation errors with different μ settings. (c) indicates the preference on the middle region of the PF, while (b) indicates the preference on an extreme.

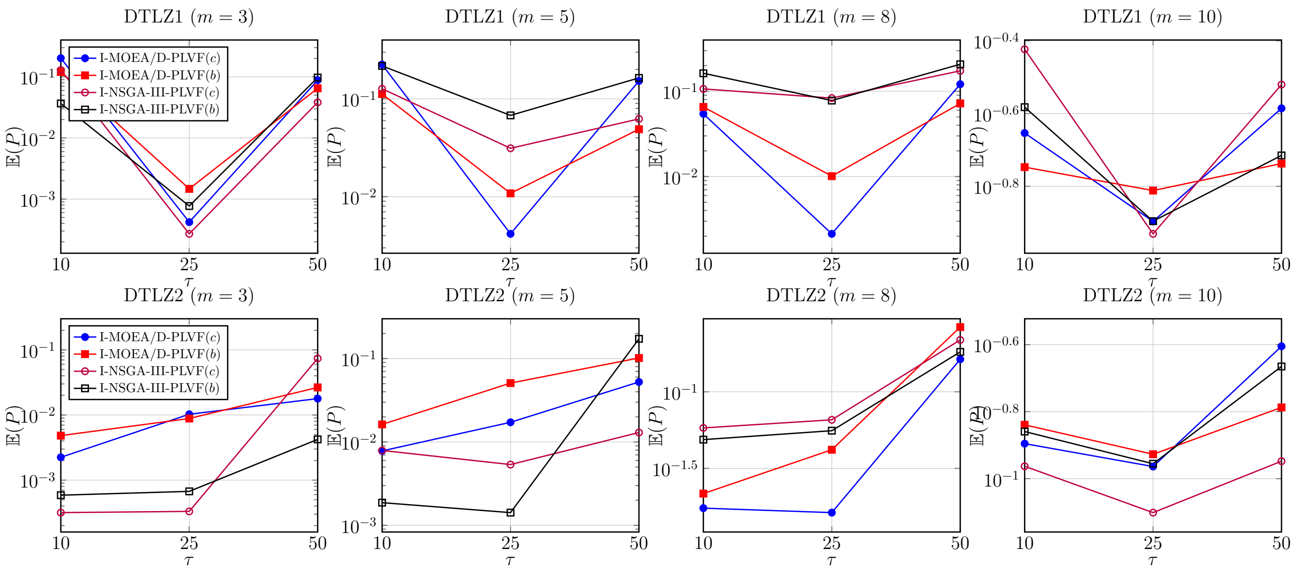


Figure 13: Variations of the approximation errors with different τ settings. (c) indicates the preference on the middle region of the PF, while (b) indicates the preference on an extreme.

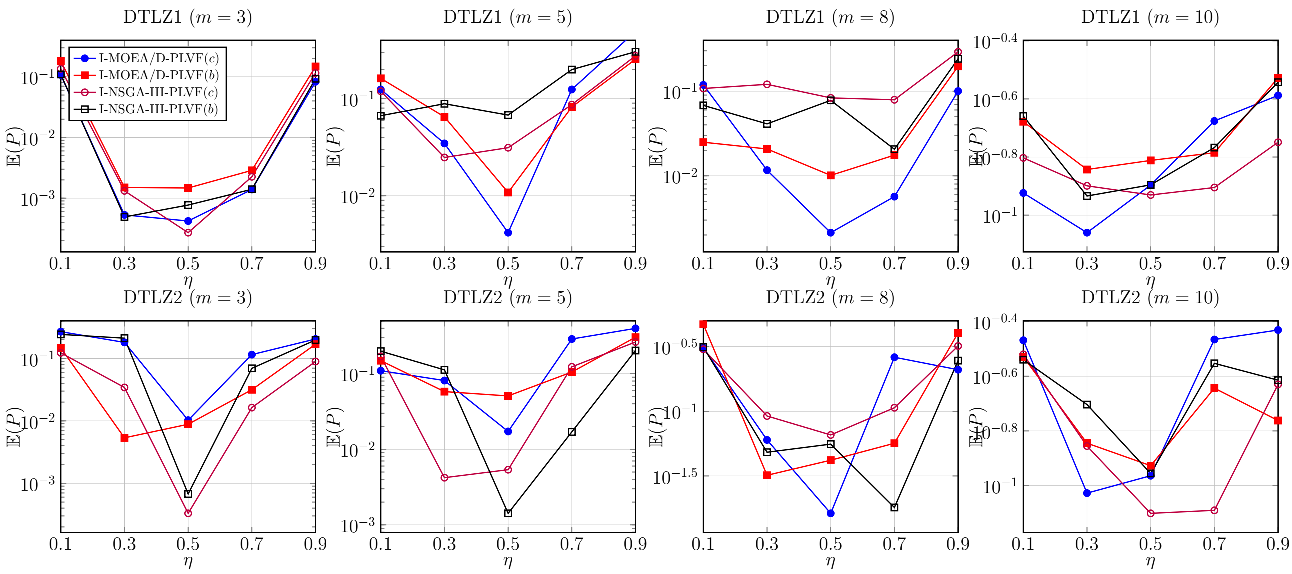


Figure 14: Variations of the approximation errors with different η settings. (c) indicates the preference on the middle region of the PF, while (b) indicates the preference on an extreme.

6 Influence of DM's Inconsistencies

In this section, we show the empirical results on different noise level κ settings. In particular, $\kappa \in \{0.0, 0.1, 0.5\}$ where $\kappa = 0.0$ means that the DM's golden value function is deterministic as usual.

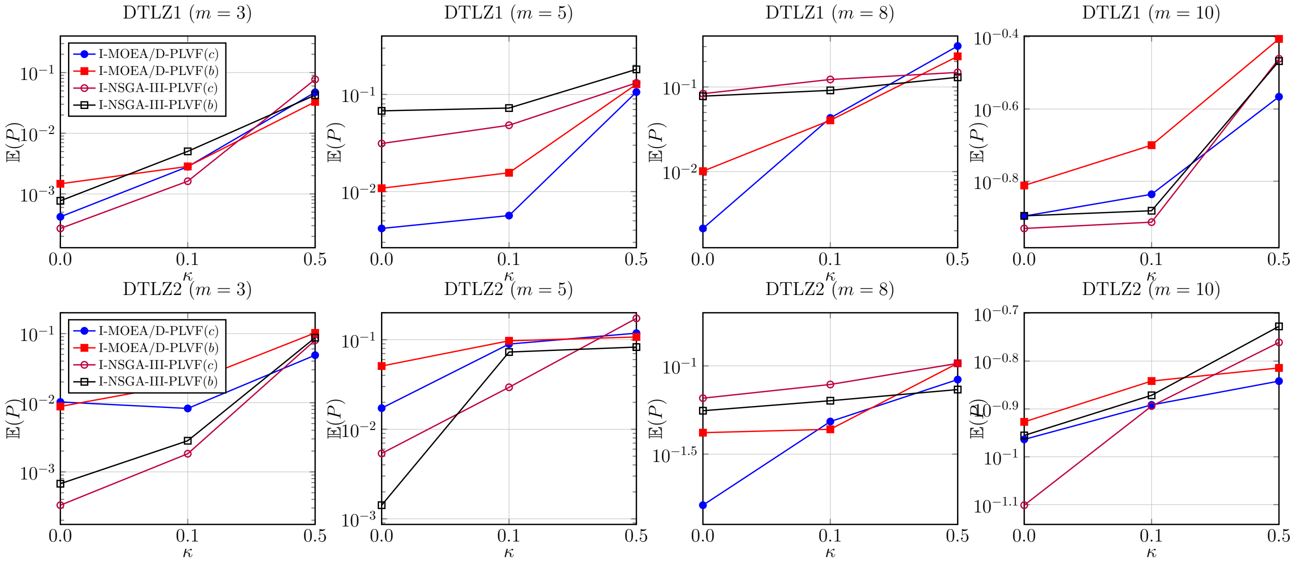


Figure 15: Variations of the approximation errors with different κ settings. (c) indicates the preference on the middle region of the PF, while (b) indicates the preference on an extreme.

Acknowledgment

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References

[1] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable test problems for evolutionary multi-objective optimization," in *Evolutionary Multiobjective Optimization*, ser. Advanced Information and Knowledge Processing, A. Abraham, L. Jain, and R. Goldberg, Eds. Springer London, 2005, pp. 105–145.